MAHALAKSHMI
ENGINEERING COLLEGE
TIRUCHIRAPALLI – 621213

DEPARTMENT: CIVIL **SEMESTER: VII**

SUB.CODE/ NAME: CE2403/ BASICS OF DYNAMICS AND ASEISMIC DESIGN

UNIT – I

THEORY OF VIBRATION

Concept of inertia and damping – Types of Damping – Difference between static forces and dynamic excitation – Degrees of freedom – SDOF idealization – Equations of motion of SDOF system for mass as well as base excitation – Free vibration of SDOF system – Response to harmonic excitation – Impulse and response to unit impulse – Duhamel integral.

Two Marks Questions and Answers

1. What is mean by Frequency?

Frequency is number of times the motion repeated in the same sense or alternatively. It is the number of cycles made in one second (cps). It is also expressed as Hertz (H_Z) named after the inventor of the term. The circular frequency ω in units of sec^{-1} is given by 2π f.

2. What is the formula for free vibration response?

The corresponding equation under free vibrations can be obtained by substituting the right hand side of equation as zero. This gives

mu + Cu +Ku = 0

3. What are the effects of vibration?

i. Effect on Human Sensitivity.
ii. Effect on Structural Damage

4. What is mean by theory of vibration?

Vibration is the motion of a particle or a body or a system of concentrated bodies having been displaced form a position of equilibrium, appearing as an oscillation.

Vibration was recognized in mechanical systems first and hence the study of vibrations fell into the heading “Mechanical Vibrations” as early about 4700 years ago.

5. Define damping.

Damping is a measure of energy dissipation in a vibrating system. The dissipating mechanism may be of the frictional form or viscous form. In the former case, it is called dry friction or column damping and in the latter case it is called viscous damping. Damping in a structural system generally assumed to be of viscous type for mathematical convenience. Viscous damped force (F_d) is proportional to the velocity ($\dot{u}$) of a vibrating body. The constant of proportionality is called the damping constant (C). Its units are NS/m.

6. What do you mean by Dynamic Response?

The Dynamic may be defined simply as time varying. Dynamic load is therefore any load which varies in its magnitude, direction or both, with time. The structural response (i.e., resulting displacements and stresses) to a dynamic load is also time varying or dynamic in nature. Hence it is called dynamic response.

7. What is mean by free vibration?

A structure is said to be undergoing free vibrations if the exciting force that caused the vibration is no longer present and the oscillating structure is purely under influence of its own inertia or mass(m) and stiffness (k). Free vibration can be set in by giving an initial displacement or by giving an initial velocity (by striking with a hammer) to the structure at an appropriate location on it.

8. What is meant by Forced vibrations?

Forced vibrations are produced in a structure when it is acted upon by the continuous presence of an external oscillating force acting on it. The structure under forced vibration normally responds at the frequency ratio, i.e. (fm/fn) where fm is the frequency of excitation and is the natural frequency of the structure.

9. Write a short note on Amplitude.

It is the maximum response of the vibrating body from its mean position.

Amplitude is generally associated with direction – vertical, horizontal, etc. It can be expressed in the form of displacement (u), velocity ( ) or acceleration ( ). In the case of simple harmonic motion, these terms are related through the frequency of oscillation (f).

If 'u' is displacement amplitude, then

Velocity ( ) = 2π f .u

Acceleration ( ) = (2π f). (u) = 4 πu

When acceleration is used as a measure of vibration, it is measured in terms of acceleration due to gravity, g (9.81 m/sec^2).

10. Define Resonance.

This phenomenon is characterized by the build –up area of large amplitudes of any given structural system and as such , it has a significance in the design of dynamically loaded structures. Resonance should be avoided under all circumstances, whenever a structure is acted upon by a steady state oscillating force (i.e., fm is constant). The presence of damping, however, limits the amplitudes at resonance. This shows the importance of damping in controlling the vibrations of structures. According to IS 1893 – 1975- Indian standard code of practice on Earthquake resistant design of structures, following values of damping are recommended for design purposes.

11. What is mean by Degrees of freedom?

The number of degrees of freedom of system equals the minimum number of independent co-ordinates necessary to define the configuration of the system.

12. Define static force.

A push or pull or a load or many loads on any system creates static displacement or deflection depending on whether it is a lumped system or a continues system; there is no excitation and hence there is no vibration.

13. Write a short note on simple Harmonic motion.

Vibration is periodic motion; the simplest form of periodic motion is simple harmonic. More complex forms of periodic motion may be considered to be composed of a number of simple harmonics of various amplitudes and frequencies as specified in Fourier series

14. What is the response for impulsive load or Shock loads?

Impulsive load is that which acts for a relatively short duration. Examples are impact of a hammer on its foundation. Damping is not important in computing response to impulsive loads since the maximum response occurs in a very short time before damping forces can absorb much energy from the structure. Therefore, only the undamped response to impulsive loads will be considered.

15. Write a short note on single degree of freedom (SDOF) systems.

At any instant of time, the motion of this system can be denoted by single co-ordinate (x in this case). It is represented by a rigid mass, resting on a spring of stiffness 'k' and coupled through a viscous dashpot (representing damping) having constant 'C'. Here, the mass 'm' represents the inertial effects of damping (or energy dissipation) in the system. Using the dynamic equilibrium relation with the inertial force included, according to D'Alembert's principle, it can be written as

$$F_I \; + \; F_D \; + F_S \; = P\,(t)$$

F_I +	F_D	+ F_S	= P (t)
(Inerti Force	(Damp force)	(Elas force	(App forc

This gives

mx + Cx +Kx = P

x, x, x respectively denote the displacement, velocity and acceleration of the system. P (t) is the time dependent force acting on the mass. The above equation represents the equation of motion of the single degree freedom system subjected to forced vibrations**.**

16. Define Cycle.

The movement of a particle or body from the mean to its extreme position in the direction, then to the mean and then another extreme position and back to the mean is called a Cycle of vibration. Cycles per second are the unit Hz.

17. Write short notes on D'Alembert's principle.

According to Newton's law F = ma

The above equation is in the form of an equation of motion of force equilibrium in which the sum of the number of force terms equal zero. Hence if an imaginary force which is equal to **ma** were applied to system in the direction opposite to the acceleration, the system could then be considered to be in equilibrium under the action of real force **F** and the imaginary force **ma**. This imaginary force **ma** is known as **inertia force** and the position of equilibrium is called **dynamic equilibrium**.

D'Alembert's principle which state that a system may be in dynamic equilibrium by adding to the external forces, an imaginary force, which is commonly known as the inertia force

18. Write the mathematical equation for springs in parallel and springs in series

Springs in parallel

$$k_e = k_1 \; k_2$$

k_e is called equivalent stiffness of the system

Springs in series

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

19. Define logarithmic decrement method.

Logarithmic decrement is defined as the natural logarithmic value of the ratio of two adjacent peak values of displacement in free vibration. It is a dimensionless parameter. It is denoted by a symbol

20. Write short notes on Half-power Bandwidth method.

Bandwidth is the difference between two frequencies corresponding to the same amplitude. Frequency response curve is used to define the half-power bandwidth. In which, the damping ratio is determined from the frequencies at which the response amplitude is reduced 1, times the maximum amplitude or resonant amplitude.

21. Define Magnification factor.

Magnification factor is defined as the ratio of dynamic displacement at any time to the displacement produced by static application of load.

22. What is the difference between a static and dynamic force?

In a static problem, load is constant with respect to time and the dynamic problem is the time varying in nature. Because both loading and its responses varies with respect to time

Static problem has only one response that is displacement. But the dynamic problem has mainly three responses such as displacement, velocity and acceleration.

23. Define critical damping.

Critical damping is defined as the minimum amount of damping for which the system will not vibrate when disturbed initially, but it will return tot the equilibrium position. This will result in non-periodic motion that is simple decay. The displacement decays to a negligible level after one nature period T.

24. List out the types of damping.

(1) Viscous Damping, **(2)** Coulomb Damping, **(3)** Structural Damping, **(4)** Active Damping, **(5)** Passive Damping.

25. What is meant by damping ratio?

The ratio of the actual damping to the critical damping coefficient is called as damping ratio. It is denoted by a symbol and it is dimensionless quantity. It ca be written as $\rho = c$

16 MARKS

EXAMPLE 2.2 A system vibrating with a natural frequency of 6 Hz starts with an initial amplitude (x_0) of 2 cm and an initial velocity ($\dot{x}_0$) of 25 cm/s. Determine the natural period, amplitude, maximum velocity, maximum acceleration and phase angle. Also write the equation of motion of a vibrating system.

Solution: **Given details:**

$$f = 6 \text{ Hz}$$
$$x_0 = 2 \text{ cm}$$
$$\dot{x}_0 = 25 \text{ cm/s}$$

The natural period is given by, $T = \frac{1}{f} = \frac{1}{6} = 0.167\text{s}$

The amplitude of motion $A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$

where, $\omega_n = 2\pi f = 2\pi(6) = 37.7 \text{ rad/s}$

$$A = \left[2^2 + \frac{25^2}{37.7^2}\right]^{1/2}$$

or $$A = 2.11 \text{ cm}$$

The maximum velocity of a system is given by

$$\dot{x}_{max} = A\omega_n = 2.11 \times 37.7$$
$$= 79.44 \text{ cm/s}$$

The maximum acceleration of a system is $\ddot{x}_{max} = A\omega_n^2 = 2.11 \times 37.7^2$

$$= 2994.76 \text{ cm/s}^2$$

Phase angle $$\phi = \tan^{-1}\left[\frac{x_0\omega_n}{\dot{x}_0}\right]$$

$$= \tan^{-1}\left[\frac{2 \times 37.7}{25}\right]$$
$$= 71°39'23''$$
$$= 1.25 \text{ rad.}$$

Equation of motion is $x = A\sin(\omega_n t + \phi) = 2.11\sin(37.7t + 1.25)$

EXAMPLE 2.3 A vertical cable 3 m long has a cross-sectional area of 4 cm² supports a weight of 50 kN. What will be the natural period and natural frequency of the system? $E = 2.1 \times 10^6$ kg/cm².

Solution: Given details:

$$A = 4 \text{ cm}^2$$
$$w = 50 \text{ kN}$$

∴ $$m = \frac{w}{g} = \frac{50 \times 10^3}{9.81} = 5096.8 \text{ kg}$$

$$E = 2.1 \times 10^6 \text{ kg/cm}^2$$

Stiffness $$k = \frac{AE}{L} = \frac{4 \times 2.1 \times 10^6}{300} = 7000 \text{ kg/cm}$$
$$= 7000 \times 981 = 6.867 \times 10^6 \text{ N/m}$$

Natural frequency, $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.867 \times 10^6}{5096.8}}$$
$$= 36.7 \text{ rad/s}$$

Natural period, $$T = \frac{2\pi}{36.7}$$
$$= 0.17 \text{ s}$$

Frequency $$f = \frac{1}{T} = 5.84 \text{ Hz (or) cps}$$

EXAMPLE 2.4 A one kg mass is suspended by a spring having a stiffness of 1 N/mm. Determine the natural frequency and static deflection of the spring.

Solution: Given details:

$$k = 1 \text{ N/mm} = 1000 \text{ N/m}$$
$$m = 1 \text{ kg}$$

Natural frequency $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000}{1}} = 31.62 \text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = 5.03 \text{ Hz}$$

Static deflection δ_{st}

We know that, $$\omega_n = \sqrt{\frac{g}{\delta_{st}}}$$

$$\delta_{st} = \frac{g}{\omega_n^2} = 9.81 \times 10^{-3} \text{ m} = 9.81 \text{ mm}$$

EXAMPLE 2.5 A cantilever beam 3 m long supports a mass of 500 kg at its upper end. Find the natural period and natural frequency. $E = 2.1 \times 10^6$ kg/cm² and $I = 1300$ cm⁴.

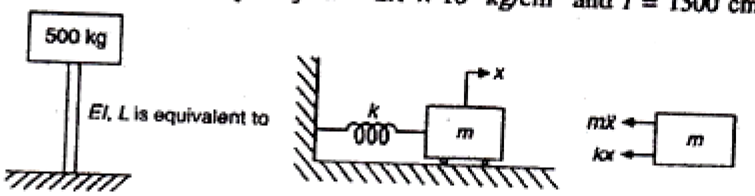

Figure 2.11

Solution:

Flexural stiffness for a cantilever beam, $k = \frac{3EI}{L^3}$, $k = \frac{3 \times 2.1 \times 10^6 \times 1300}{(300)^3}$

$$= 303 \text{ kg/cm} = 303 \times 981 \text{ N/cm}$$
$$= 2.97 \times 10^5 \text{ N/cm}$$

Natural frequency $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.97 \times 10^5}{500}}$

$$= 24.37 \text{ rad/s}$$

or $f = \frac{\omega_n}{2\pi} = 3.88 \text{ cps}$

Natural period $T = \frac{1}{f} = \frac{2\pi}{\omega_n} = 0.26 \text{ s}$

EXAMPLE 2.6 A cantilever beam AB of length L is attached to a spring k and a mass M as shown in Figure 2.12. (a) Form the equation of motion; and (b) Find an expression for the frequency of motion.

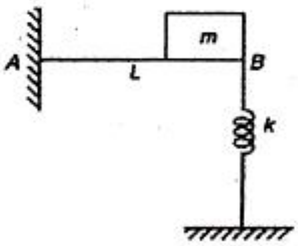

Figure 2.12

Solution:

(a) Equation of motion

Stiffness due to the applied mass M is $k_1 = \frac{M}{\Delta} = \frac{3EI}{L^3}$

This stiffness k_1 is acting parallel to k

$\therefore$ Equivalent spring stiffness $k_e = k_1 + k$

$$= \frac{3EI}{L^3} + k = \frac{3EI + kL^3}{L^3}$$

The differential equation of motion is

$$m\ddot{x} = -k_e x$$

$$\Rightarrow \qquad m\ddot{x} + k_e x = 0$$

$$\Rightarrow \qquad m\ddot{x} + \left[\frac{3EI + kL^3}{L^3}\right]x = 0$$

$$\Rightarrow \qquad \ddot{x} + \left[\frac{3EI + kL^3}{L^3 m}\right]x = 0$$

(b) The frequency of vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k_e}{m}} = \frac{1}{2\pi}\sqrt{\frac{kL^3 + 3EI}{mL^3}}$$

EXAMPLE 2.7 Find the natural frequency of the system as shown in Figure 2.13. Take $k_1 = k_2 = 2000$ N/m, $k_3 = 3000$ N/m and $m = 10$ kg.

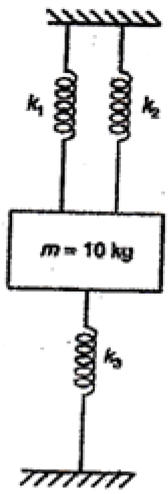

Figure 2.13

Solution: The equivalent system is shown in Figure 2.14.

Two springs k_1 and k_2 are in parallel

Equivalent stiffness $k_{e1} = k_1 + k_2 = 2000 + 2000$

$= 4000$ N/m

Again this equivalent spring is parallel to k_3

$\therefore$ Equivalent stiffness, $k_e = k_{e1} + k_3 = 4000 + 3000$

$= 7000$ N/m

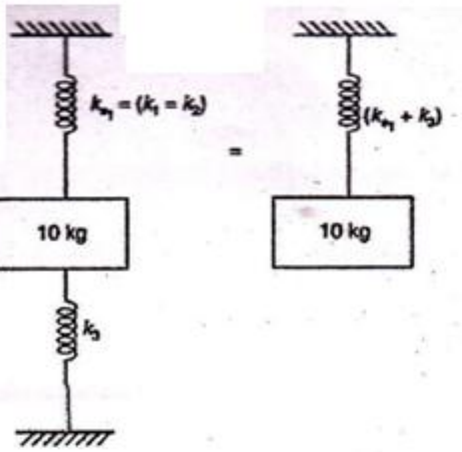

Figure 2.14

Natural frequency,
$$\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{7000}{10}}$$
$$= 26.46 \text{ rad/s}$$

or
$$f = \frac{\omega_n}{2\pi} = 4.21 \text{ Hz}$$

EXAMPLE 2.8 Consider the system shown in Figure 2.15.

If $k_1 = 2000$ N/m, $k_2 = 1500$ N/m, $k_3 = 3000$ N/m and $k_4 = k_5 = 500$ N/m, find the mass if the system has a natural frequency of 10 Hz.

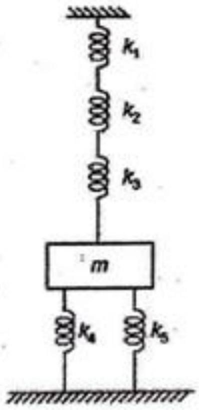

Figure 2.15

Solution: Given details:

$$k_1 = 2000 \text{ N/m}, \quad k_2 = 1500 \text{ N/m}$$
$$k_3 = 3000 \text{ N/m}, \quad k_4 = k_5 = 500 \text{ N/m}$$
$$f = 10 \text{ Hz.}$$

The springs k_1, k_2 and k_3 are in series. Their equivalent stiffness

$$\frac{1}{k_{e1}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{2000} + \frac{1}{1500} + \frac{1}{3000}$$

or $$k_{e1} = 666.67 \text{ N/m.}$$

The two lower springs k_4 and k_5 are connected in parallel, so their equivalent stiffness

$$k_{e2} = k_4 + k_5 = 500 + 500 = 1000 \text{ N/m}$$

Again these two equivalent springs are in parallel,

$$k_e = k_{e1} + k_{e2} = 666.67 + 1000$$

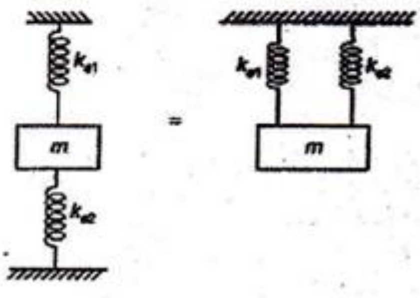

Figure 2.16 Equivalent spring.

Figure 2.16 Equivalent spring.

$\Rightarrow$ $$k_e = 1666.67 \text{ N/m}$$

$$f = \frac{\omega_n}{2\pi}$$

$\Rightarrow$ $$\omega_n = 2\pi f = 2\pi(10)$$

$\Rightarrow$ $$\omega_n = 62.83 \text{ rad/s}$$

But $$\omega_n = \sqrt{\frac{k}{m}}$$

$\Rightarrow$ $$\omega_n^2 = \frac{k}{m}$$

$\Rightarrow$ $$m = \frac{k_e}{\omega_n^2} = \frac{1666.67}{(62.83)^2}$$
$$= 26.52 \text{ kg}$$

EXAMPLE 2.9 Determine the natural frequency of the system shown in Figure 2.17.

Solution:

For a cantilever beam, the stiffness is $k_b = \dfrac{3EI}{L^3} = \dfrac{3\times200\times10^9\times1.6\times10^{-5}}{4^3}$

$$= 1.5 \times 10^5 \text{ N/m}$$

Now the beam and the spring k_1 are acting parallel. This combination is in series with k_2. This series combination is in parallel with k_3 and k_4.

$$\therefore \quad k_{e1} = k_3 + k_4 = 4\times10^5 + 6\times10^5$$

$$= 10 \times 10^5 \text{ N/m}$$

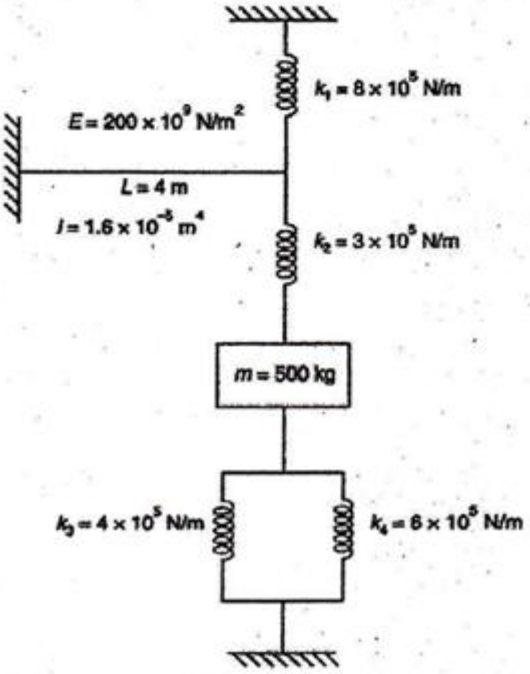

Figure 2.17

But $$k_e = \frac{1}{\left(\dfrac{1}{k_b+k_1}\right)+\dfrac{1}{k_2}} + k_{e1} = \frac{1}{\left(\dfrac{1}{1.5\times10^5+8\times10^5}\right)+\dfrac{1}{3\times10^5}} + (10\times10^5)$$

$$\Rightarrow \quad k_e = 12.28 \times 10^5 \text{ N/m.}$$

Natural frequency $\omega_n = \sqrt{\dfrac{k_e}{m}} = \sqrt{\dfrac{12.28\times10^5}{500}}$

$$= 49.56 \text{ rad/s}$$

EXAMPLE 2.10 A simply supported rectangular beam has a span of 1 m. It is 100 mm wide and 10 mm deep. It is connected at mid-span of a beam by means of a linear spring having a stiffness of 100 kg/cm and a mass of 300 kg is attached at the other end of spring. Determine the natural frequency of the system. Take $E = 2.1 \times 10^6$ kg/cm^2.

Solution:

The stiffness of simply supported beam is $k_b = \frac{48EI}{L^3}$

$$I = \frac{bd^3}{12} = \frac{10(1)^3}{12} = 0.833 \text{ cm}^4$$

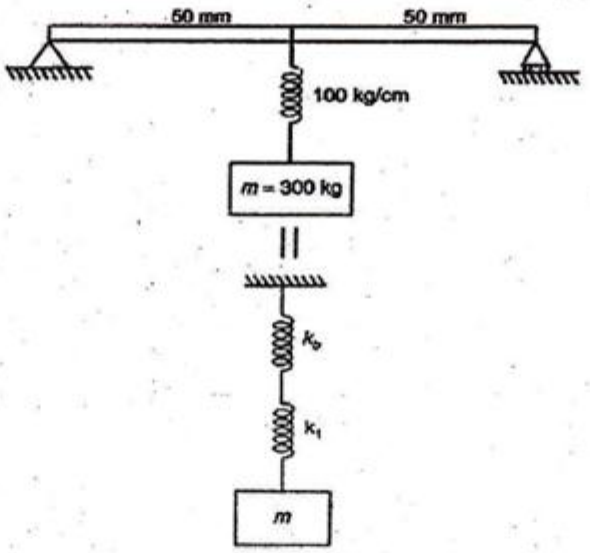

Figure 2.18

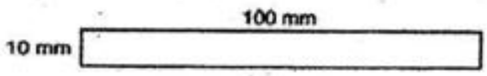

Figure 2.19

$$k_b = \frac{48 \times 2.1 \times 10^6 \times 0.833}{(100)^3}$$

$$= 84 \text{ kg/cm}$$

The two springs k_1 and k_2 are in series, the equivalent stiffness of the compound spring is given by

$$\frac{1}{k_e} = \frac{1}{k_b} + \frac{1}{k_1} = \frac{1}{84} + \frac{1}{100}$$

$\Rightarrow$ $$k_e = 45.65 \text{ kg/cm}$$

$$= 45.65 \times 981 = 0.448 \times 10^5 \text{ N/cm}$$

Natural frequency $\omega_n = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{0.448 \times 10^5}{300}}$

$$= 12.22 \text{ rad/s}$$

Logarithmic Decrement Method

This method is used to measure damping in time domain. In this method, the free vibration displacement amplitude history of a system is measured and recorded.

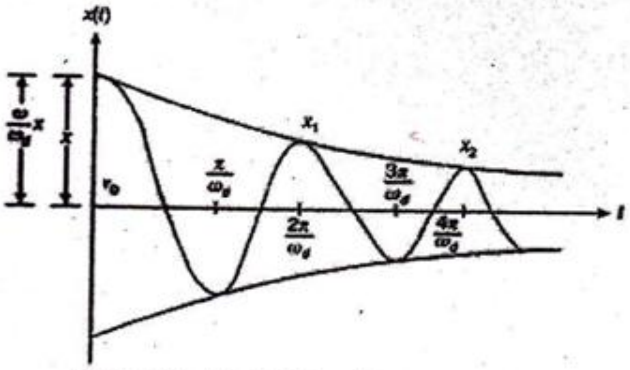

Figure 3.11 Amplitude decay for viscous damping.

shows the amplitude decay curve. Logarithmic decrement is defined as the natural logarithmic value of the ratio of two adjacent peak values of displacement in free vibration. It is a dimensionless parameter. It is denoted by a symbol δ.

$$\delta = \ln\frac{x_1}{x_2}$$

In case of underdamped system, the general solution is given by

$$x = Xe^{-\rho\omega_n t}\left(\sin\omega_d t + \phi\right)$$

Let the displacement within one cycle is x_0

$$\text{Amplitude } x_0 = xe^{-\rho\omega_n t}$$

Let x_1 is the displacement after one cycle

$$\text{Amplitude } x_1 = xe^{-\rho\omega_n}(t + T_d)$$

$$\Rightarrow \quad \frac{x_0}{x_1} = \frac{xe^{-\rho\omega_n t}}{xe^{-\rho\omega_n}(t + T_d)}$$

$$= e^{-\rho\omega_n T_d}$$

Taking logarithm on both sides

$$\log\frac{x_0}{x_1} = \rho\omega_n T_d$$

$$\delta = \rho\omega_n T_d = \frac{c}{c_c}\omega_n T_d = \frac{c}{2m\omega_n}\omega_n T_d$$

$$\Rightarrow \quad \delta = \frac{c}{2m}T_d$$

where

$$T_d = \frac{2\pi}{\omega_n\sqrt{1-\rho^2}}$$

$$\Rightarrow \quad \delta = \frac{c}{2m}\frac{2\pi}{\omega_n\sqrt{1-\rho^2}} = \frac{c}{c_c}\frac{2\pi}{\sqrt{1-\rho^2}}$$

$$\Rightarrow \quad \delta = \frac{2\pi\rho}{\sqrt{1-\rho^2}}$$